UNIVERSITY OF
LINCOLN

SCHOOL OF MATHEMATICS AND PHYSICS

Lincoln Mathematics Challenge 2017/18

Submit your typed or neatly written solutions of the following problems to **maths@lincoln.ac.uk** or by post to **Mathematics Challenge, School of Mathematics and Physics, University of Lincoln, Lincoln, LN6 7TS.** Please include your full name, postal address and email, as well as the name and address of your school. The closing date is 10 January, 2018. The prize-giving ceremony will be held in Lincoln on 7 February 2018. It is possible to win a prize even if you have not completed all of the questions, so you are encouraged to submit solutions if you do only some of the problems. The competition is open to all young pre-university people in UK aged 15–18 years. It is not open to current university students. See full Terms and Conditions at `https://lincolnmathsphys.wordpress.com/challenges-terms-and-conditions`.

1. The four walls of a house in the city of Brasilia face exactly North, West, South and East. Which wall is illuminated by the Sun at exactly noon local time on 22 June? on 22 September? on 22 December?

2. Prove that for any positive real numbers a, b, c

(a) the inequality $(a+b)\Big(\dfrac{1}{a}+\dfrac{1}{b}\Big) \geqslant 4$ holds;

(b) the inequality $(a+b+c)\Big(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\Big) \geqslant 9$ holds.

3. In a right-angled triangle ABC with $\angle C = 90°$, the length of AC is 20 cm and the radius of the inscribed circle is 5 cm. Find the length of BC.

4. Prove that it is impossible to change some of the signs "+" to "−" in the sum

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2016} + \frac{1}{2017} + \frac{1}{2018}$$

in such a way that the result would become equal to 0.

5. Positive integers are written on cards in the following pattern: 1 is written on the 1st card, then 2 is written on the 2nd and 3rd cards, then 3 is written on the next three cards, then 4 is written on the next four cards, and so on. Which number will be written on the 2017th card?

6. Given 21 cities, some of them are connected by direct flights. Each of these 21 cities is assigned a rank equal to the number of these flight connections from this city. It turned out any two cities with the same rank are not connected. What is the maximum possible number of connections under this restriction?

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.