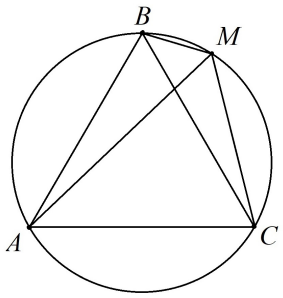
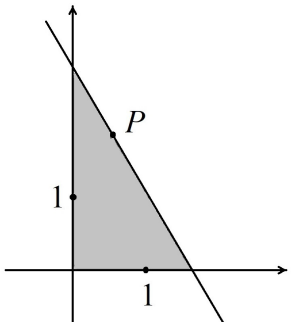
1. Solve the system of simultaneous equations $\begin{cases} y^2 = 4^x + 8 \\ 2^{x+1} + y + 1 = 0 \end{cases}$.

2. Let $\triangle ABC$ be an equilateral triangle, and M an arbitrary point on the circumference of the circumscribed circle. Prove that the sum of distances from M to two of the points A, B, C is equal to the distance from M to the third of these points. (In the picture, $MA = MB + MC$.)



3. Find all integers z such that the number $z^2 + 4z + 16$ is a square of an integer, giving reasons for your answer.

4. For which points P in the first quadrant of the coordinate plane does there exist a straight line through P such that the triangle formed by this line and the coordinate axes has area 2? (Picture not to scale.)



5. Prove that among any 100 consecutive positive integers there is a number such that the sum of its digits is divisible by 14.

6. On the circumference of a circle, 4036 points are chosen dividing the circumference into 4036 equal arcs. These points are connected in pairs by 2018 chords. (Thus, each of these points is an endpoint of exactly one of the chords; the chords may intersect one another.) Prove that among these chords at least two have the same length.

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.