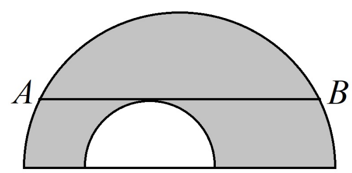
1. Find the values of x that simultaneously satisfy all three inequalities

$$\begin{cases} x^2 - 6x + 8 \geqslant 0 \\ x^2 - 9x + 20 \geqslant 0 \\ x^2 - 6x + 5 \leqslant 0. \end{cases}$$

2. One semicircle lies inside another semicircle as on the picture, so that their diameters are on the same straight line. The tangent to the smaller semicircle parallel to the diameter intersects the larger semicircle in points A and B.

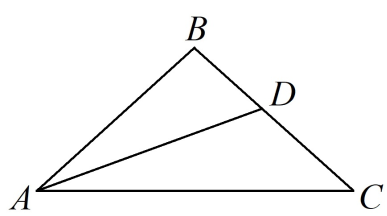


Given that the length of AB is 7 cm, find the shaded area within the larger semicircle outside the smaller one (that is, bounded by two pieces of the diameter, the larger semicircle and the smaller one).

3. Let A be a positive integer with n digits, and a its right-most digit. Let B be the integer with $n-1$ digits obtained from A by erasing its right-most digit a. Prove that A is divisible by 7 if and only if $B + 5a$ is divisible by 7.

4. Let $a_1 < a_2 < \cdots < a_{2019}$ be 2019 strictly increasing positive integers. Prove that the least common multiple of all these integers is not less than $2019 \cdot a_1$.

5. In the triangle ABC the angles are $\angle BAC = \angle ACB = 40°$. Let AD be the bisector of $\angle BAC$ with D on the side BC. Prove that the length of AC is equal to the sum of the lengths of BD and AD.



6. Prove that any fraction $\frac{a}{b}$, where $a < b$ are positive integers, can be represented as a sum of the form

$$\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$$

for some n and some pairwise different positive integers $k_1, k_2, \ldots, k_n$.

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.