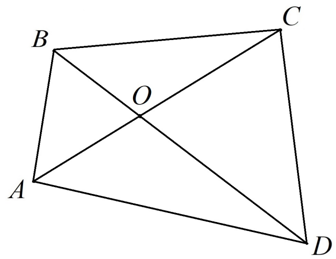
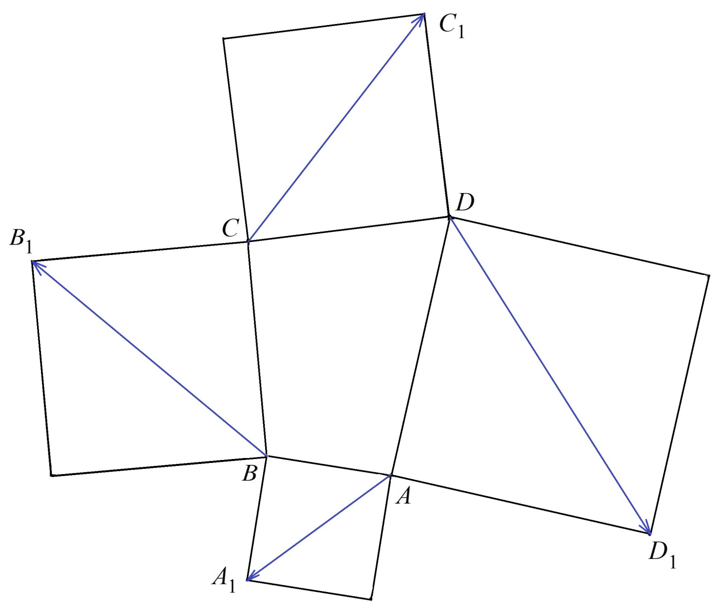
1. Let O be the intersection point of the diagonals AC and BD of a convex quadrangle $ABCD$. Determine the minimum possible area of $ABCD$ under the condition that the area of $\triangle ABO$ is $25\,\mathrm{cm}^2$ and the area of $\triangle CDO$ is $49\,\mathrm{cm}^2$.



2. Solve the system of simultaneous equations

$$\begin{cases} x^2 + xy + y^2 = 4 \\ x^4 + x^2y^2 + y^4 = 8. \end{cases}$$

3. Let $ABCD$ be an arbitrary convex quadrangle. On every side of $ABCD$, squares are constructed on the outside of $ABCD$ as on the picture. Let A_1, B_1, C_1, D_1 be the vertices of these squares opposite to the vertices A, B, C, D, respectively. Prove that the sum of vectors $\overrightarrow{AA_1}$, $\overrightarrow{BB_1}$, $\overrightarrow{CC_1}$, $\overrightarrow{DD_1}$ is equal to $\vec{0}$.



4. Represent the following product in the form of a reduced fraction $\dfrac{m}{n}$ with coprime positive integers m, n and find the sum $m+n$:

$$\frac{2^3-1}{2^3+1} \cdot \frac{3^3-1}{3^3+1} \cdot \frac{4^3-1}{4^3+1} \cdot \frac{5^3-1}{5^3+1} \cdots \frac{2020^3-1}{2020^3+1}.$$

5. The positive integers 1, 2, 3, …, 2020 are placed on a circle in some order. Prove that one can always find three neighbouring numbers (that is, three numbers appearing consecutively on the circle) the sum of which is not divisible by 5.

6. On the alphabet of two letters A, B, one forms 2^n words (arbitrary sequences composed of the letters A, B) such that none of these words is an initial segment of any other of these words. (An initial segment of a word can be of any length, from a single first letter, to the whole word.) Prove that the sum of lengths of these words is at least $n \cdot 2^n$.

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.