

1. Suppose that in the year 2021 fossil fuel power generation, nuclear power stations, and renewable sources contributed to overall electricity generation in the ratio of 11:4:10, respectively. If in the year 2022 the fossil fuel power generation decreases by 5%, and nuclear increases by 2%, by how much must the renewable power generation increase in percentage in order to maintain the same level of overall electricity generation?
2. In a triangle ABC , let I denote the centre of the inscribed circle, and O the centre of the circumscribed circle. Given that $\angle AIO = 90^\circ$ and $\angle CIO = 45^\circ$, find the ratios of the lengths of the sides $AB : BC : CA$.
3. If the sum of digits of a positive integer A is 2021, and the sum of digits of a positive integer B is 2022, what is the minimum possible sum of digits of the number $A + B$?
4. A small grasshopper starts jumping from a point O on the plane. It makes its first jump of length 1 cm (along some straight line), then the second jump of length 2 cm in either of the two directions perpendicular to the first jump, then the third jump of 3 cm in either of the two directions perpendicular to the second, and so on, each time increasing the length of the jump by 1 cm and choosing one of the two directions perpendicular to the preceding jump.
 - (a) Can the grasshopper return to the origin O after 71 jumps?
 - (b) Can the grasshopper return to the origin O after 2022 jumps?
5. A regular 2021-gon is inscribed in a circle with centre O . Each vertex of the 2021-gon is coloured with one of three colours blue, green, or red, and each of these colours is used for at least one of the vertices. Prove that it is always possible to choose three vertices, one blue, one green, and one red, such that the triangle formed by connecting these three vertices contains the centre O .
6. Positive integers $1, 2, 3, \dots, 2022$ are arbitrarily divided into three groups each containing 674 numbers. Prove that it is always possible to choose three numbers, one from each group, such that one of these numbers is the sum of the other two.

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.