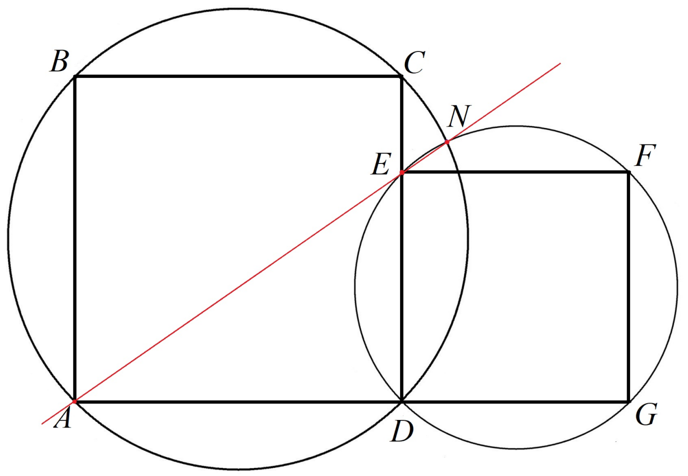
1. Two squares $ABCD$ and $DEFG$ have a common vertex D, while the vertex E of the second square is on the side CD of the first, as shown on the picture. Prove that the straight line through the points A and E passes through the intersection point N of the circles circumscribed about these squares.



2. Consider the following infinite triangular table:

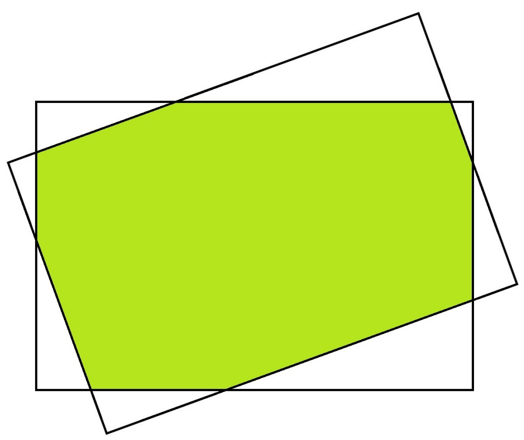
					1					
				1	1	1				
			1	2	3	2	1			
		1	3	6	7	6	3	1		
	1	4	10	16	19	16	10	4	1	
.	.	.	.	.	.	.	.	.	.	.

The first row consists of a single 1. All subsequent rows are calculated as follows: every number is the sum of the three entries adjacent to it in the row above (where blank entries count as zero). For example, the three adjacent entries above 4 are 0, 1 and 3, and of course $4 = 0+1+3$. Prove that every row, starting from the third one, contains an even number.

3. If x, y, z are positive integers such that $x + y + z = 2023$, what is the maximum possible value of $xy + xz + yz$?

4. The integers $1, 2, 3, \ldots, 2022$ are rearranged in some order $a_1, a_2, \ldots, a_{2022}$ in such a way that the sum of any two consecutive numbers is at most 2512, that is, $a_i + a_{i+1} \leqslant 2512$ for every $i = 1, 2, \ldots, 2021$. Prove that there is j such that $a_j + a_{j+2} > 2512$.

5. Two congruent rectangles are situated on the plane in such a way that their sides intersect in eight points as shown on the picture. Prove that the area of the common part of these rectangles is at least half the area of one of these rectangles.



6. Prove that for any sequence of 2023 positive real numbers $r_1, r_2, \ldots, r_{2023}$ one can find a positive integer $k \leqslant 2023$ such that each of the k numbers

$$r_k, \quad \frac{r_k + r_{k-1}}{2}, \quad \frac{r_k + r_{k-1} + r_{k-2}}{3}, \quad \ldots \quad , \frac{r_k + r_{k-1} + \cdots + r_1}{k}$$

is not greater than $\dfrac{r_1 + r_2 + \cdots + r_{2023}}{2023}$.

(It is also possible that $k = 1$, when those k numbers consist of only r_k, or $k = 2$, when those k numbers consist of only r_k and $(r_k + r_{k-1})/2$.)

Note: Full solutions are required — not just answers — with complete proofs of any assertions you may make.